

# Measurement of the $K^- \pi^+ S$ -wave System in $D^+ \rightarrow K^- \pi^+ \pi^+$ Decays from Fermilab E791

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**Abstract.** A new approach to the analysis of three body decays is presented. Measurements of the  $S$ -wave  $K\pi$  amplitude are made in independent ranges of invariant mass from threshold up to the upper kinematic limit in  $D^+ \rightarrow K^- \pi^+ \pi^+$  decays. These are compared with results obtained from a fit where the  $S$ -wave is assumed to have  $\kappa$  and  $K_0^*(1430)$  resonances. Results are also compared with measurements of  $K^- \pi^+$  elastic scattering. Contributions from  $I = \frac{1}{2}$  and  $I = \frac{3}{2}$  are not resolved in this study. If  $I = \frac{1}{2}$  dominates, however, the Watson theorem prediction, that the phase behaviour below  $K\eta'$  threshold should match that in elastic scattering, is not well supported by these data. Production of  $K^- \pi^+$  from these  $D$  decays is also studied.

## INTRODUCTION

Decays of  $D$  and  $B$  mesons show promise as a source of information on the light-quark mesons they produce. Their decays to  $S$ -wave systems in three pseudo-scalar final states, may help to improve our knowledge of the particularly confusing scalar meson ( $J^P = 0^+$ ) spectrum. Until now, extracting such information has been done in model-dependent ways that make assumptions about the scalar states observed that can influence the results. With large, clean samples of such decays, anticipated to be coming from the  $B$  factories and the Tevatron collider, the need for new approaches is a priority.

Knowledge of strange scalar mesons has relied on measurements of  $S$ -wave  $K^- \pi^+$  scattering. These come principally from SLAC experiment E135 (LASS) [1] and cover the invariant mass range above 825 MeV/ $c^2$ . Data from other experiments below this range exist, but with less precision [2]. More measurements in the low mass region are required if the possibility of the existence of a  $\kappa$  state is to be properly evaluated.

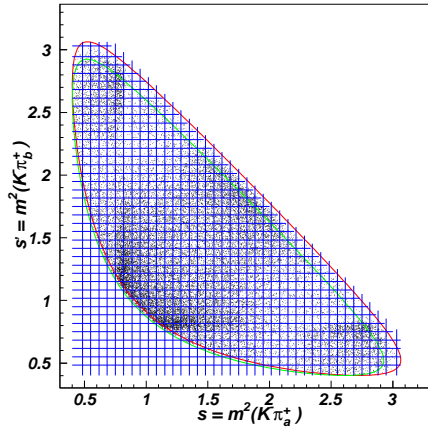
In this paper, a study of the decays [Note]  $D^+ \rightarrow K^- \pi^+ \pi^+$  observed in data from Fermilab experiment E791 is presented. In an earlier analysis [3] the  $K^- \pi^+ S$ -wave was modelled with Breit-Wigner (BW) amplitudes for  $\kappa$  and  $K_0^*(1430)$  resonances. A less model-dependent analysis is presented here.  $S$ -wave amplitudes are measured without assuming any specific dependence on  $K^- \pi^+$  invariant mass, or on the presence of scalar resonances. Results are compared with the amplitude from our earlier analysis, and also with measurements from LASS.

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## DATA SAMPLE

The selection process for events used in this paper is described in Ref. [3]. A signal consisting of 15,079  $D^+ \rightarrow K^- \pi_a^+ \pi_b^+$  decays, with a purity of  $\sim 94\%$ , is obtained. Fig. 1 shows the Dalitz plot with the  $K^- \pi_a^+$  invariant mass squared plotted vs. that for the  $K^- \pi_b^+$  system. Horizontal (and symmetrized vertical) bands corresponding to the  $K^*(892)$  resonance are clearly seen. A complex pattern of both constructive and destructive interference is seen near 2  $(\text{GeV}/c^2)^2$  due to presence of  $K_0^*(1430)$ ,  $K_1^*(1410)$  and  $K_2^*(1430)$ . Evidence for  $K_1^*(1680)$ , difficult to see due to smearing of the Dalitz plot boundary resulting from the finite resolution in the three-body  $D^+$  mass, may also exist.



**FIGURE 1.** Dalitz plot for  $D^+ \rightarrow K^- \pi_a^+ \pi_b^+$  decays. The squared invariant mass of one  $K^- \pi^+$  combination is plotted against the other combination. The plot is symmetrized, each event appearing twice. Lines in both directions indicate values, equally spaced in squared effective mass, at each of which the  $S$ -wave amplitude is determined by the method described in the text. Kinematic boundaries for the Dalitz plot are drawn for three-body mass values  $M = 1.810$  and  $M = 1.890 \text{ GeV}/c^2$ , between which data are selected.

An asymmetry along the  $K^*(892)$  bands, most easily described by interference with a significant  $S$ -wave component to these decays, is also observed. Information on this  $S$ -wave amplitude is obtained from its interference with the  $K^*(892)$ , and also the other well-established resonances in the Dalitz plot.

## METHOD

In Ref. [3], as in most earlier analyses of  $D$  decays to three pseudo-scalar particles  $ijk$ , the isobar model, with BW resonance forms, is used. The decay amplitude  $\mathcal{A}$  is described by a sum of quasi two-body terms  $D \rightarrow R + k$ ,  $R \rightarrow i + j$ , in each of the three channels  $k = 1, 2, 3$ :

$$\mathcal{A} = d_0 e^{i\delta_0} + \sum_{n=1}^N d_n e^{i\delta_n} \frac{F_R(p, r_R, J)}{m_{R_n}^2 - s - i m_{R_n} \Gamma_{R_n}(s)} \times F_D(q, r_D, J) M_J(p, q) \quad (1)$$

The squared invariant mass of the  $ij$  system is  $s$ ,  $J$  is the spin,  $m_{R_n}$  the mass and  $\Gamma_{R_n}(s)$  the width of each of the  $N$  resonances  $R_n$  seen to be contributing to the decay. For  $J > 0$ ,  $F_R$  and  $F_D$  are Blatt-Weisskopf form factors [4], with effective radius parameters  $r_R$  and  $r_D$ , for all  $R_n$  and for the parent  $D$  meson, respectively. For  $J = 0$ , a Gaussian form suggested by Tornqvist [5] is used for the  $D$ . The momenta,  $\vec{p}$  and  $\vec{q}$  for  $i$  and  $k$ , respectively, are defined in the  $ij$  rest frame, and  $M_J(p, q) = (-2pq)P_J(\hat{p} \cdot \hat{q})$  is introduced to describe spin conservation in the decay. The complex coefficients  $d_n e^{i\delta_n}$  ( $n = 0, N$ ) depend on the  $D$  decay and are determined by a fit to the data. The first term describes non-resonant ( $NR$ ) decay to  $i + j + k$  with no intermediate resonance, and is assumed to be independent of  $s$ . For  $D^+ \rightarrow K^- \pi_a^+ \pi_b^+$  decays we Bose-symmetrize  $\mathcal{A}$  with respect to interchange of  $\pi_a^+$  and  $\pi_b^+$ .

In Ref. [3], an excellent fit to the data is obtained with  $\kappa$  and  $K_0^*(1430)$  resonant terms and the  $NR$  term comprising the  $J = 0$  part of Eq. (1).

In this paper, the  $K^- \pi^+$   $S$ -wave is examined in a less model-dependent way, outlined here. A more detailed account is given in [6].

Terms appearing in Eq. (1) are grouped according to the value of  $J$ . The  $S$ -wave part (all terms with  $J = 0$ , including the  $NR$  term) is factored

$$\mathcal{S} = S(s) \times M_0(p, q) F_D(q, r_D, 0) \quad (2)$$

into a partial wave  $S(s)$ , describing  $K^- \pi^+$  scattering, and the product  $M_0(p, q) F_D(q, r_D)$  describing the  $D$  decay. The  $P$ - and  $D$ - ( $J = 1, 2$ , respectively) waves are factored in the same way:

$$\mathcal{P} = P(s) \times M_1(p, q) F_D(q, r_D, 1); \quad \mathcal{D} = D(s) \times M_2(p, q) F_D(q, r_D, 1), \quad (3)$$

with partial waves  $P(s)$  and  $D(s)$  consisting of resonant terms given as in Eq. (1). The  $S$ -wave  $S(s)$ , however, is replaced by a set of values  $c_k e^{i\gamma_k}$  defined at 40 discrete squared invariant masses  $s = s_k$ . These are indicated by the lines in Fig. 1. The  $c_k$  and  $\gamma_k$  values are regarded as independent parameters to be determined by the data.

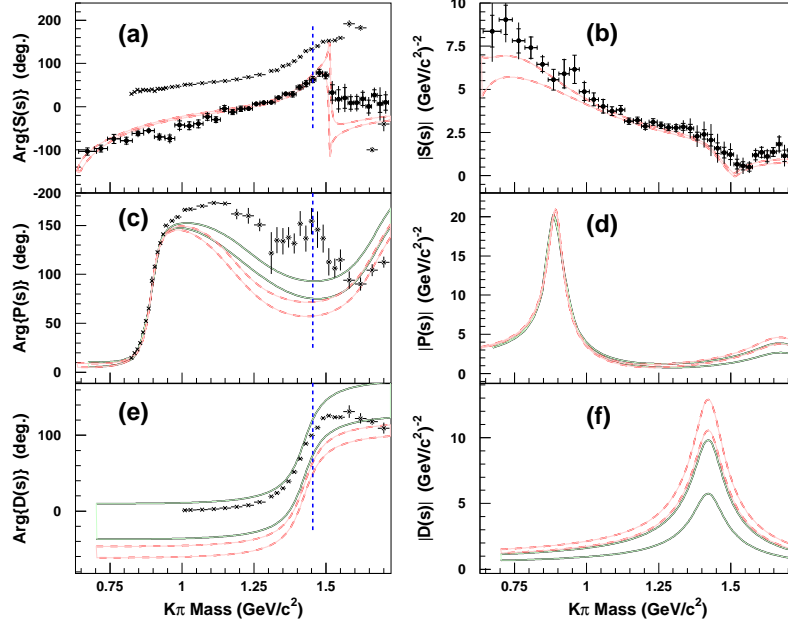
An unbinned likelihood fit is made to the data, in a similar way to that described in Ref. [3]. An incoherent function describing the 6% background in the sample from events that are not true  $D$  decays is added in proper proportion at each three-body  $K^- \pi^+ \pi^+$  mass to a signal distribution proportional to  $|\mathcal{A}|^2$ . There are 86 parameters - all  $(c_k, \gamma_k)$  and the coefficients  $d_k e^{i\delta_k}$  for  $K^*(892)$ ,  $K_1^*(1680)$  in the  $P$ -wave and  $K_2^*(1430)$  in the  $D$ -wave. For the  $K^*(892)$ ,  $d_k e^{i\delta_k} = 1$  is used to provide the reference amplitude.

## RESULTS AND COMPARISON WITH MODEL-DEPENDENT FIT

This fit also results in an excellent description of the data. Comparison of the observed and predicted population of the Dalitz plot gives a  $\chi^2$  probability of 50% for 363 bins. The  $S$ -,  $P$ - and  $D$ -waves resulting from the fit are shown in Fig. 2.

These results are compared with the model-dependent fit from Ref. [3] which provided an excellent description of the observed Dalitz plot distribution. Partial waves for this fit are also shown in Fig. 2. The main  $S$ -wave features of both fits agree well. Some

differences, particularly at the highest and lowest ends of the invariant mass range, result from shifts in the  $P$ - and  $D$ -waves. Resonant fractions and the total  $S$ -wave fraction (about 75%) also agree well within statistical limits.



**FIGURE 2.** (a) Phases  $\gamma_k = \arg S(s_k)$  and (b) magnitudes  $c_k = |S(s_k)|$  of  $S$ -wave amplitudes for  $K^-\pi^+$  systems from  $D^+ \rightarrow K^-\pi^+\pi^+$  decays with the amplitude and phase of the  $K^*(892)$  as reference. Solid circles, with error bars, show the values obtained from the model-independent fit described in the text. The effect of adding systematic uncertainties in quadrature is indicated by extensions on the error bars. The magnitudes plotted include a  $D^+$  form-factor  $F_D(q, r_D, 0) = e^{-25q^2/12}$ . The phase and magnitude of  $P(s)$  are shown in (c) and (d), respectively. In each case, the parameters and error matrix from the fit are used in Eq. (3) to produce the solid curves shown one standard deviation above and below the central values. Similar plots for the  $D$ -wave amplitude  $D(s)$  are shown in (e) and (f). In all plots, the dashed curves show similar, one standard deviation limits for the amplitudes obtained from the isobar model fit in Ref. [3]. In (a), (c) and (e),  $I = \frac{1}{2}$  phases for  $K^-\pi^+$  scattering measured in the LASS experiment are shown as  $\times$ 's with error bars indicating statistical uncertainties.

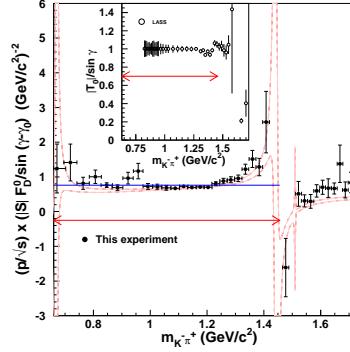
It can be concluded that, with the present sample size, no significant distinction between the model-dependent and model-independent parametrizations of the  $S$ -wave can be made.

A comparison of the  $S$ -wave amplitudes  $S(s)$  measured here with the amplitudes  $T(s)$  obtained in  $K^-\pi^+$  scattering is now made. For each partial wave  $J$  (for each isospin  $I$ ) it is expected that  $S(s) = Q(s)T(s)/F_D(q, r_D, J)\sqrt{s}/p^{(J+1)}$  where  $Q(s)$  describes the  $s$ -dependence of  $K^-\pi^+$  production in  $D$  decays. The Watson theorem [7] requires, provided there is no re-scattering of the  $K^-\pi_a^+$  from  $\pi_b^+$ , that  $Q(s)$  is a real function, so that phases found in  $D$  decay should match those in  $K^-\pi^+$  elastic scattering data.

$I = 1/2$  phases measured by LASS are plotted in Fig. 2. There is a large offset in the  $S$ -wave, about  $75^\circ$ , not seen in  $P$ - or  $D$ -waves. The shapes of  $S$ - and  $P$ -waves are also not the same. Unless significant admixture of  $I = 3/2$   $K^-\pi^+$  production occurs, these

results suggest that the conditions for the Watson theorem are not met in these data.

The production function  $Q(s)$  is shown in Fig. 3. The scattering amplitude  $T(s)$  is assumed to be elastic,  $T = \sin(\gamma - \gamma_0)$ . The phase offset  $\gamma_0$  is required to account for the difference between elastic scattering and  $D$  decay. There appears to be significant  $s$ -dependence above  $\sim 1.2 \text{ GeV}/c^2$  that grows in the region near  $1.4 \text{ GeV}/c^2$  where  $\gamma \sim \gamma_0$ .



**FIGURE 3.** The quantities  $p/\sqrt{s_k} \times |S(s_k)| \times F_D(q, r_D, 0)/\sin(\gamma_k - \gamma_0)$  plotted as solid circles for each point obtained for the  $S$ -wave amplitude in the model-independent fit described in the text. Three points between 1400 and 1450  $\text{MeV}/c^2$  are omitted from the plot as their values for  $\sin(\gamma_k - \gamma_0)$  are very small, making their errors extremely large. The region between the dashed lines shows the one standard deviation limits of this quantity for the  $S$ -wave amplitude obtained from the model-dependent fit. The inset shows, as small open circles, the quantities  $|T_0(s)|/\sin(\gamma)$  taken from the LASS experiment. The elastic range, up to  $K\eta'$  threshold, is indicated by double-headed arrows in both plots.

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- Note. Note, Charge conjugate states are always implied unless explicitly stated otherwise.
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